Mugberia Gangadhar Mahavidyalaya

Department of Mathematics

Differential Equations

Mathematics (Hons.): Sem-I(2019): Full Marks 31

Any seven from Group -A: $3 \times 7 = 21$

1. Show that the differential equation of all parabolas with foci at the origin and axis along x-axis is given by
$y(\frac{dy}{dx})^2 + 2x\frac{dy}{dx} - y = 0$ [V.U.2002]

2. Solve : $(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdy = 0$ [V.U.2002]

3. Reduced the differential equation $x^2p^2 + py(2x + y) + y^2 = 0$ to Clairaut's form by the substitutions $y = u,\ xy = v$, solve it for singular solution and extraneous loci, if any.

4. Show that the substitution $x = e^u$ transforms the equation
$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \cos x$ into $\frac{d^2y}{du^2} + 3\frac{dy}{du} + 2y = \cos x$. JAM(MA)–2010

5. Prove that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$ is
$(x^2 - 2xy - y^2 + 1)dx + (x^2 + 2xy - y^2 - 1)dy = 0$ **V.U(Hons.)-2017**

6. Explain the terms: general solution, a particular solution, a singular solution as applied to an ordinary differential equation.

7. The equation
$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, \quad \textbf{WBSSC 2001} \quad (1)$$
(where a and b are fixed constants and λ is an arbitrary parameter which can assume all real values) represents a family of confocal conics. To obtain the differential equation of this family.

8. If $\frac{1}{M-N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x + y)$, then the differential equation $Mdx + Ndy = 0$ has an integrating factor of the form $e^{-\int f(x+y)d(x+y)}$.

9. Show that the general solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ can be written in the form $y = k(f - g) + g$ where k is an arbitrary constant and f, g are its particular solutions. BU(H) 2010, CU(H) -2009

10. Solve the problem $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$.

11. Solve

$$(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

12. Solve the differential equation $x\frac{dy}{dx} + y = y^2 \log x$

Answer any two from the Group -B: $2 \times 5 = 10$

1. Let M, N be two real-value functions which have continuous first partial derivatives on some rectangle

$$R: \ |x - x_0| \leq a, \ |y - y_0| \leq b, \quad (a, b > 0).$$

Then the necessary and sufficient conditions for the ordinary differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact in R is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ in } R.$$

2. Reduce the differential equation $(px^2 + y^2)(px + y) = (p+1)^2$ to Clairaut's form by the substitutions $u = xy, v = x + y$ and then obtain the complete primitive. **C.H.-92; V.H-00.**

3. **Prove that if $Mx + Ny \neq 0$ and the equation $Mdx + Ndy = 0$ be homogeneous differential equation where M, N have continuous first partial derivatives on some rectangle R, then $\frac{1}{Mx+Ny}$ in R is an integrating factor of the said equation. V.U(H) : 2016**

4. **Show that the general solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ can be written in the form $y = \frac{Q}{P} - e^{-\int Pdx}\left[e^{\int Pdx} d\left(\frac{Q}{P}\right) + c\right]$ where c is an arbitrary constant. V.U(H) : 2017**